The method of automatic suitable simulation for spray systems

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Abstract

In this paper, according to system connection, a branch network model of pipes and components is set up for the spray system actual piping condition. The recursive algorithm is commencement at the end of branch pipe where the input is the distal elbow upstream operating pressure head. It is end on the inlet of main pipe, where we get the requirement of total head for the pipe's system, from being compared with the pump thrust under the same discharge. The approximation of system's total head can be given out by the formula of Kalman Filter.

Base on the theorem of Model Reference Adaptive Control System quoted from the Modern Control Theory, the Method of Standard Reference Adaptive Control was introduced to meet the demands of research and analysis of Sprinkling Irrigation Pipeline Network. This theorem greatly improved the accuracy of hydraulic analysis and can give out all properties of the piping network system.

Keywords

Spray system, Math model, Simulation, Hydraulic analysis, Pipeline network, Adaptive Control System.

Introduction

As well know, when we do the hydraulic analysis of the spray system. We normally assume that sprayer operating pressure and outlet flow are considered as constant, usually accepted with the design working pressure and design flow of sprayers. But in many situations, this assume is not work because of the topography and the variation of pump. In this article the hydraulic analysis of spray system can be improved for more accuracy and for more efficient utility of water and power. Because it uses a new method.

The mathematics model of spray system

A. The model of pipe system

1. The model of sprayer

In the system of spray, the model of sprayer (micro-sprinkler or jet) can be given out by the digit model quoted from the characteristic curve of the sprayer or by the formula from the mathematical statistics which mostly given as

$$q = aH^b \qquad (1)$$

Where ***q*** is the outlet flow of the sprayer, ***a*** is the coefficient for the sprayer, ***H*** is the operating pressure of the sprayer and ***b*** is the pressure head ***H*** exponent. The ***a, b*** can be obtained from the test of the sprayer.

2. The model of the lateral

As we know, the Bernoulli's Theorem states the sum of the water head on the cross-section as following

$$h = \frac{p}{r} + \frac{v^2}{2g} + z$$

Where ***h*** is the total water head of the cross-section,***p/r*** is the pressure head, ***v^2/2g*** is the velocity head, ***z*** is the pipe center elevation of the cross-section.

$$h_{i,j} = hf_{j-1} + h_{i,j-1} = \sum_{k=j-1}^{m_i} hf_k + h_{i,m_i} \qquad (2)$$

$$hf_k = \frac{F_k L_k Q_{i,k}^m}{D_k^n} + \frac{K_k Q_{i,k}^2}{2gA_k^2} \qquad (3)$$

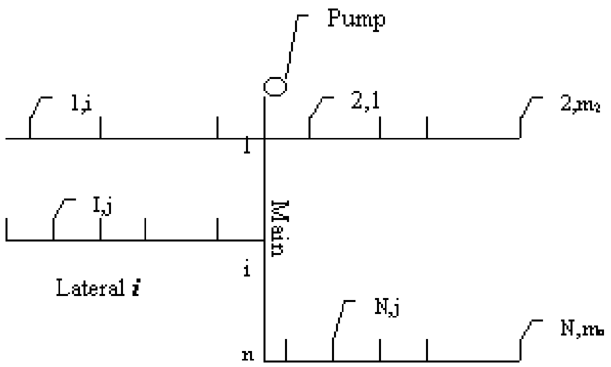


Fig. 1

Where ***$h_{i,j}$*** is the ***i***th branch pipe's of ***j***th cross-section total head. ***hf_k*** is the ***k***th section head loss along the pipe in ***i***the branch. ***F_k*** is a friction factor along the pipe. ***L_k*** or ***D_k*** is the ***k*** section length or the diameter of the pipe section. ***A_k***is the cross-section area of the pipe. ***K_k*** is the local resistance coefficient for fittings and couplers. ***$Q_{i,k}$*** is the flow in ***k***th pipe section.

The discharge ***$Q_{i,k}$*** can be calculated by the follow equation

$$Q_{i,k} = \sum_{p=k}^{m_i} q_{i,p} \qquad (4)$$

Where the sprayer outlet flow ***$q_{i,p}$*** comes from the equation (1). It is

$$q_{i,p} = a(h_{i,p} - r_{i,p})^b \qquad (5)$$

where

$$r_{i,p} = e_{i,p} + \frac{u_{i,p}^2}{2g} + y_{i,p} \qquad (6)$$

Here ***$e_{i,p}$*** is the elevation of the sprayer. ***$u_{i,p}^2$/2g*** is the velocity head of the riser. ***$y_{i,p}$*** is the head loss along the riser. From Eq. (2), (3), (4) and (5), it is easy to understand that the nd distal pressure head acts as an argument for the function of ***$h_{i,j}$***. Here let ***s_i=$h_{i,mi}$***, to start at the last elbow upstream cross-section pressure ***s_i*** and work back to the upstream of branch we get the function

$$h_{i,0} = h_{i,0}(s_i) \tag{7}$$

Here $h_{i,o}$ should be emphasized that the friction loss hf_1 is included the loss in the valve and the tee, which coupled the branch with the main line.

3. The model of main pipe line

Considering spray system as show in Fig. 1, the discharge of the main pipe's *i*th section

$$Q_i = \sum_{k=i}^{n} Q_{k,1}(s_k) = Q_i(s_i, s_{i+1} \dots s_n) \tag{8}$$

Here $Q_{k,1}$ is the flow in the inlet of the *k*th branch pipe. At the inlet or first section of main line the discharge Q presents as

$$Q = \sum_{k=1}^{n} Q_{k,1}(s_k) = Q(s_1, s_2 \dots s_n) \tag{9}$$

From the input s_i to the upstream of the main line, the computation gives out the total head which required by *i*th branch

$$H_i = \sum_{k=1}^{i} hf_k(Q_k) + h_{i,0}(s_i) = H_i(s_1, s_2 \dots s_n) \tag{10}$$

Where $hf_k(Q_k)$ is the function of discharge Q_k that presents water head loss along *k*th section of the main pipe.

B. The pump model

A numeral model of pump quoted from the characteristic curve of pump with the Lagrange Interpolation Formula, or simple using the following model for it [1] .

$$H_0 = AN^2 + BNQ - CQ^2$$

or just as below

$$H_0 = H_0(Q) = H_0(s_1, s_2 \dots s_n) \tag{11}$$

The adaptive principle for the spray system

Let's consider the following single dimension discrete-time system which expressed as

$$H(k+1) = H(k) + U(k) \tag{12}$$

$$\mathrm{H}_0(k) = H(k) + V(k) \tag{13}$$

Where $H(k)$ is the total head of the spray system. k is the iterative number. $U(k)$ is the adjustable parameter of the total head, which is unequal to zero till the steady condition. $H_0(k)$ is the total head criterion comes from the pump called standard reference of total head. $V(k)$ is the adjustable parameter reflecting difference between $H_0(k)$ and $H(k)$. Assuming that the equations (12) and (13) would obey with the condition of the Kalman Filter Formula, we get the formulations
for the spray network system

$$H^*(k+1) = H^*(k) + K(k+1)(H_0(k+1) - H^*(k)) \tag{14}$$

$$\mathrm{K(k+1) = P(k+1/k)(P(k+1/k) + R(k+1))^{-1}} \tag{15}$$

$$\mathrm{P(k+1/k) = P(k) + G(k)} \tag{16}$$

$$\mathrm{P(k) = (1 - K(k))P(k/k-1)} \tag{17}$$

$$\mathrm{for\ k = 0,1,2\dots\ \dots}$$

Where $H^*(k)$ is the estimate for the total water head $H(k)$. $K(k)$ is Kalman Filter gain on *k*th iterative computation. ***P(k+1/k)*** is the covariance of the error coming from the estimation, which expresses the difference between ***H(k+1)*** and ***H^*(k)***. ***P(k)*** is the covariance about the error between ***H(k)*** and ***H^*(k)***. ***G(k)*** and ***R(k)*** are the variances of the adjustable parameters ***U(k)*** and ***V(k),*** which are independent zero mean noises obeying Gaussian sequence condition. Here the initial condition is

$$H^*(0) = \frac{1}{n}\sum_{i=1}^{n} H_i(0), \quad P(0) = P$$

Where ***P*** is a constant. In Eq. (15) let ***k*** replace ***k+1***, we get

$$K(k) = P(k/k-1)(P(k/k-1) + R(k))^{-1}$$
$$\text{or } K(k)R(k) = (1 - K(k))P(k/k-1)$$

compared with Eq. (17) yielding to

$$K(k)R(k) = P(k)$$

Above equation was substituted into Eq. (16) and (15), resulted in below

$$K(k+1) = (K(k)R(k) + G(k))(K(k)R(k) + G(k) + R(k+1))^{-1} \qquad (18)$$

Now, if we got the value of ***K(0)***, ***G(k)*** and ***R(k)***, then the Kalman gain ***K(k)*** would be progressively obtained.

Real filtering procedure

1. The determination of K(0), G(k) and R(k)

Here one can use the method quoted from the references [3] and [4] for identification of the noise characteristics in Kalman Filter. Since the system and measurement equation in here are expressed as scalar, we use the following way to substitute the complicated one. About the value of ***K(0)***, the reference [6] theorem 2 has been pointed out that for the complete controllability and observability system, the Kalman gain ***K(0)*** would not depend on the initial value of ***P(0)***, while the calculating recursion for a long time. But ***K(0)*** is related with ***P(0)***, therefor, in the prior unknown condition,
it is convenience to let ***K(0)***=1.

For the residuals ***U(k)*** and ***V(k)***, a sampling plan should be brought forward to approximate the variances ***G(k)*** and ***R(k)***. Due to the statistical analysis, the pipe system output messages ***H_i(k)*** (***i=1,2 ... n***) are the system total head ***H(k)'***s ***n*** samples. According to the ergodic theorem the statistical characteristics of ***H_i(k)*** can represent for that of ***H(k)***. On the other hand, ***H_0(k)*** obtained from the pump called as the standard reference head of the model, can be used as the measurement of ***H(k)***. The new information

$$e_i(k) = H_i(k) - H_{i+1}(k) \qquad (\text{for } i = 1,2\ldots n-1)$$
$$e_n(k) = H_n(k) - H_1(k)$$
$$r_i(k) = H_0(k) - H_i(k) \qquad (\text{for } i = 1,2\ldots n)$$

Where ***e_i(k)*** is the sample of ***U(k)***. ***r_i(k)*** is the sample of ***V(k)***. From those samples, the variance statistic results ***G*(k), R*(k)*** are the estimations of ***G(k), R(k).*** Only let

$$\frac{G^*(k)}{R^*(k)} \geq \frac{G(k)}{R(k)}$$

We can ensure that will be convergent for the filtering [6] .

2. Really filtering formula

When the variation of ***G(k)*** and ***R(k)*** is smooth, let's suppose

$$G(k) \le G^*(k), \quad R(k) \le R(k+1) \le R^*(k)$$

then Eq. (18) has been reformed as

$$K(k+1) = (K(k)R^*(k) + G^*(k))(K(k)R^*(k) + G^*(k) + R^*(k))^{-1} = 1 - (K(k) + K_p + 1)^{-1} \quad (19)$$

where ***Kp =G*(k)/R*(k).*** Using the Eq. (14), Eq. (19) and adding with the initial condition

$$H^*(0) = \frac{1}{n}\sum_{i=1}^{n} H_i(0), \qquad K(=10)$$

We can reconstruct a recursive Kalman Filter Formula.

3. One step filtering method

In the several beginning step of filtering, the assumption of zero mean white noise processes that may not be obeyed. Then the one step filtering method should be derived as below

$$H^*(k+1) = \overline{H} + K(k+1)(H_0(k+1) - \overline{H}) \qquad (20)$$

$$K(k+1) = 1 - (K(k) + K_p + 1)^{-1} \qquad (21)$$

here $\overline{H} = \frac{1}{n}\sum_{i=1}^{n} H_i(k).$

The simulating experience shows that when ***k =N***

$$\left|H_0(N) - H_i(N)\right| \le r \qquad \text{for } i = 1,2\ldots n$$

being tenable, then from ***k=N+1,*** the recursive computation starts to use Eq. (14) and (19). The value of ***r*** can be given out by experiment.

4. The filtering formula for sprayer discharge

The similar way for the estimation of the sprayer's outlet flow is given out by following equation

$$q^*_{i,p}(k+1) = q^*_{i,p}(k) + K(k+1)(q_{i,p}(k+1) - q^*_{i,p}(k)) \qquad (22)$$

where the measurement for

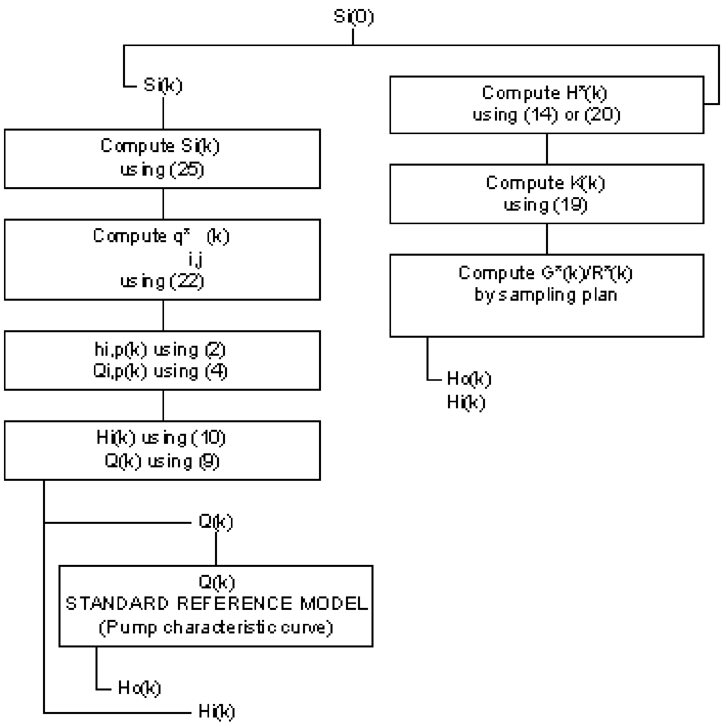
$$q_{i,p}(k+1) = a(h_{i,p}(k) - r_{i,p}(k))^b \qquad (23)$$

$h_{i,p}(k)$ comes from Eq. (2). $r_{i,p}(k)$ obtained by $q_{i,p}*(k)$ using the Eq. (6).

Since the state variable of outlet flow and its measurement is all most the same, in other word, the ratio Kp=1 then

$$K(k+1) = 1 - (K(k) + 2)^{-1} \qquad (24)$$

Fig.2



Adaptive control system

The result of above study has given out the approximation for total head of pipeline system, it should be used for feedback to adjust the input ***$s_i(k)$*** called as control argument. ***$s_i(k)$*** being proposed as

$$s_i(k+1) = s_i(k) + (H^*(k+1) - H_i(k)) \qquad (25)$$

thus, a standard reference adaptive control system, closed-loop, can be set up as showed in Fig. 2. Since the iteration here is similar to the Jacobi method quoted from the Iterative Solution of Nonlinear Equation, then the iteration is proved to be convergence by the principle of the reference [7] theorem 13.5.2.

Conclusion

Base on the theorem of Model Reference Adaptive Control System quoted from the Modern Control Theory, the Method of Standard Reference Adaptive Control was introduced to meet the demands of research and analysis of pipe network of spray irrigation. The Method used mathematical models of sprayer, lateral, main pipes and pump to setup a pipeline network of spray system model according
to the system's connecting condition. This theorem greatly improved the accuracy of hydraulic analysis and can give out all properties of the piping network system.

The mathematics method within this paper can be used to other hydraulic or gas delivery system, because it discusses the common solution of the piping network and gives out hydraulic analysis method to simulate the whole network properties.

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